For Supervisor's use only

3

90638

NEW ZEALAND QUALIFICATIONS AUTHORITY
MANA TOHU MĀTAURANGA O AOTEAROA

National Certificate of Educational Achievement
TAUMATA MĀTAURANGA Ā-MOTU KUA TAEA

Level 3 Calculus, 2006

90638 Manipulate real and complex numbers, and solve equations

Credits: Five
9.30 am Wednesday 29 November 2006

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

Make sure you have a copy of the Formulae and Tables booklet L3-CALCF.

You should answer ALL the questions in this booklet.

Show ALL working for ALL questions.

If you need more space for any answer, use the page(s) provided at the back of this booklet and clearly number the question.

Check that this booklet has pages 2–7 in the correct order and that none of these pages is blank.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.

For Assessor's use only	**Achievement Criteria**		
Achievement	**Achievement with Merit**	**Achievement with Excellence**
Manipulate real and complex numbers, and solve equations. ☐	Solve more complicated equations. ☐	Solve problem(s) involving real or complex numbers. ☐
Overall Level of Performance ☐		

You are advised to spend 40 minutes answering the questions in this booklet.

Assessor's use only

QUESTION ONE

(a) Write $\frac{2}{3+\sqrt{7}}$ with a rational denominator.

(b) p and q are complex numbers, where $p = 2 + 5\text{i}$ and $q = 6 - \text{i}$.

Find pq, expressing your answer in rectangular form, $a + b\text{i}$.

(c) Write $\left(6\text{cis}\frac{\pi}{12}\right)^4$ as a complex number in rectangular form, $a + b\text{i}$.

QUESTION TWO

Assessor's use only

(a) Solve $4^{7x-1} = 12$.

(b) Solve $\log_7(2x - 3) = 4.5$

QUESTION THREE

Solve the following equation: $\sqrt{x + 5} = 2x - 3$.

QUESTION FOUR

Assessor's use only

Solve $z^3 = 64i$, where z is a complex number.

Write your answers in polar form, $r\,\text{cis}\theta$.

QUESTION FIVE

Assessor's use only

One root of the equation $z^3 - 10z^2 + 37z + p = 0$, where p is real, is $z = 3 + 2i$.

Find the value of p and the other two roots.

Note that Question Six follows on the next page.

QUESTION SIX

Assessor's use only

Find the locus of the point representing z if $\frac{z+\mathrm{i}}{z-\mathrm{i}}$ is purely imaginary.

Extra paper for continuation of answers if required.
Clearly number the question.

Assessor's use only

Question number	